

Part One: Probability Distributions

- Reminders:
- ★ Probability distributions list all possible outcomes (numerical) and the probability of each outcome
  - ★ Each probability must be between 0 and 1 (inclusive) and the sum of all probabilities must be 1
  - ★ The expected value is the sum of all possible outcomes times their probability

1) Determine whether the following are probability distributions. If they are, find the expected value. If they are not, state why.

a) Yes

x	3	6	9	12	15
P(x)	4/9	2/9	1/9	1/9	1/9

expected value: 6.67

b) No

x	20	30	40	50
P(x)	1.1	0.2	0.9	0.3

- can't have a probability over 1
- all probabilities must have a sum of 1

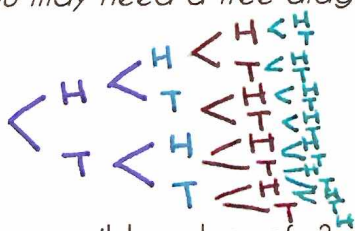
c) Yes

x	1	2	3	4	5
P(x)	3/10	1/10	1/10	2/10	3/10

expected value: 3.1

2) You flip four coins. Let  $x$  be the number of heads on all four coins.

a) List the sample space for the experiment. (What are all the possible outcomes of flipping four coins - you may need a tree diagram to help)



- HHHH<sup>4</sup> HTHH<sup>3</sup> THHH<sup>3</sup> TTHH<sup>2</sup>  
 HHHT<sup>3</sup> HTHT<sup>2</sup> THHT<sup>2</sup> TTHT<sup>1</sup>  
 HHTH<sup>3</sup> HTTH<sup>2</sup> THTH<sup>2</sup> TTTH<sup>1</sup>  
 HHTT<sup>2</sup> HTTT<sup>1</sup> THTT<sup>1</sup> TTTT<sup>0</sup>

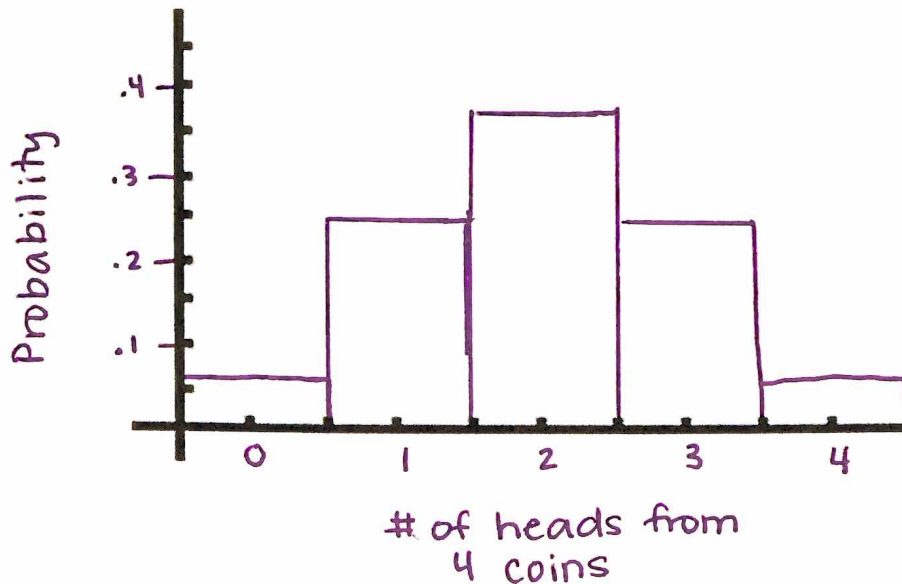
b) What are the possible values of  $x$ ?

4, 3, 2, 1, 0 or 0, 1, 2, 3, 4

c) Construct a probability distribution for this experiment.

x	0	1	2	3	4
P(x)	$\frac{1}{16}$ (.06)	$\frac{4}{16}$ (.25)	$\frac{6}{16}$ (.38)	$\frac{4}{16}$ (.25)	$\frac{1}{16}$ (.06)

d) Construct a histogram for the probability distribution.



3) Construct a probability distribution for the following data:

a) The probabilities that a patient will have 0, 1, 2, or 3 medical tests performed on entering a hospital are  $\frac{6}{15}$ ,  $\frac{5}{15}$ ,  $\frac{3}{15}$ , and  $\frac{1}{15}$  respectively.

x	0	1	2	3
P(x)	$\frac{6}{15}$	$\frac{5}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

b) A die is loaded in such a way that the probabilities of getting 1, 2, 3, 4, 5, and 6 are 0.5, 0.18, 0.08, 0.08, 0.08, 0.08 respectively.

x	1	2	3	4	5	6
P(x)	0.5	0.18	0.08	0.08	0.08	0.08

c) A box contains 3 \$1 bills, 2 \$5 bills, 1 \$10 bill, and 1 \$20 bill. 7 total bills

x	1	5	10	20
P(x)	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

3 BBB GBB 2  
 2 BBG GBG 1  
 2 BGB GGB 1  
 1 BGG GGG 0

d) A family has three children. Let  $X$  represent the number of boys.

$x$	0	1	2	3
$P(x)$	$\frac{1}{8}$ (.125)	$\frac{3}{8}$ (.375)	$\frac{3}{8}$ (.375)	$\frac{1}{8}$ (.125)

4) Below is a probability distribution for the number of math failures of MHS students.

$X$	0	1	2	3	4
$P(X)$	.41	.38	.11	.08	.02

Determine the following:

a)  $P(x = 2)$

0.11

b)  $P(x < 2)$

0.79

c)  $P(x \leq 1)$

0.79

d)  $P(x > 2)$

0.1

e)  $P(x = 3 \text{ or } x = 4)$

0.1

f) What is the expected number of math failures?

0.92

### Part Two: Counting Principle, Permutations, and Combinations

Reminders: ★ *Permutations: A permutation is an arrangement of objects, without repetition, and order being important.*

★ *Combinations: A combination is an arrangement of objects, without repetition, and order not being important.*

★ *If you have a problem with repetition, you must use the multiplication counting principle*

1) Katrina plans on wearing one necklace, one bracelet, and one ring. If she owns 6 necklaces, 8 bracelets, and 5 rings, how many different jewelry combinations can she make?

$$6 \cdot 8 \cdot 5$$

$$\text{or } 6C_1 \cdot 8C_1 \cdot 5C_1$$

240

2) Kayla has 10 books in her room. She only has room for 5 of them on her shelf. Determine the number of ways that she can put the books on the shelf?

permutation

$${}_{10}P_5 = 30,240$$

3) Choosing 6 numbers, in any order, from the numbers 1 through 59 for a lottery game where the jackpot is won for all six numbers match the winning numbers.

combination

$${}_{59}C_6 = 45,057,474$$

4) Twenty students are in the tennis tournament. Determine the number of ways that they can take 1st or 2nd place.

permutation

$${}_{20}P_2 = 380$$

5) A 10 person bowling team consists of juniors and seniors. There are 20 juniors and 30 seniors in the school. If the team is to consist of 4 juniors and 6 seniors, how many different teams can be formed?

combination

$$\underbrace{{}_{20}C_4}_{\text{juniors}} \cdot \underbrace{{}_{30}C_6}_{\text{seniors}} = 2,876,839,875$$

6) At a film festival, there are eight different films that will be shown. In how many different orders can these films be shown?

permutation

$${}_8P_8 = 40,320$$

7) There are 8 different statistics books, 6 different geometry books, and 3 different trigonometry books being considered for next year. In how many ways can a textbook committee select one of each book?

$$8 \cdot 6 \cdot 3 = 144$$

8) The call letters of a radio station must have four letters. The first letter must be a K or a W. How many different call letter combinations are possible if letters may be repeated?

$$\underbrace{2}_{\text{K or W}} \cdot \underbrace{26 \cdot 26 \cdot 26}_{\text{any letter}} = 35,152$$

9) In how many different ways can the Harry Potter series of books (7 books total) be arranged in a row on a shelf?

permutation

$${}_7P_7 = 5,040$$

10) How many different four-digit ID tags can be made if it must start with a 7 and no repeats are allowed?

$$\frac{1 \cdot 9 \cdot 8 \cdot 7}{7} = 504$$

other digits - no repeats

11) How many 3-digit numbers can be formed from the digits 2, 3, 4, 5, 7 and 9?

$$\underline{6} \cdot \underline{6} \cdot \underline{6} = 216$$

12) A recently formed music group has four original songs they can play. They are asked to perform two songs at a music festival. We wish to compute the number of song arrangements the group can offer in concert.

permutation  $4P_2 = 12$

13) In a certain country, there are two political parties. Each party is responsible for nominating both a presidential and vice-presidential candidate. The candidates will participate in a debate once they are chosen. In the first party, there are 6 candidates available and in the second party there are 5 candidates available. How many different debate combinations are possible?

permutations  $\frac{6P_2}{1st\ party} \cdot \frac{5P_2}{2nd\ party} = 600$

14) Your closet contains 10 different styles of shoes. In how many ways can you pick out five different styles of shoes for the school week if you don't care which day of the week you wear each style?

combination  $10C_5 = 252$

15) In how many ways can a jury of 12 people be selected from a pool of 12 men and 10 women?

combination  $22C_{12} = 646, 646$

16) We have 5 plants, and we want to water two of them. In how many ways can we choose the plants?

combination  $5C_2 = 10$

Part Three: Binomial Distributions

$$P(x = k) = \binom{n}{k} (p)^k (1-p)^{n-k}, \text{ where } \binom{n}{k} = {}_n C_k$$

- Reminders:
- ★ There must be a fixed number of trials ( $n$ )
  - ★ Each trial is independent of one another
  - ★ The probability of success ( $p$ ) is constant over time
  - ★ There are only 2 possible outcomes: success or failure

1) An art collector has 10 valuable paintings. He protects each one with a separate alarm system which is 95% reliable. A burglar enters the premises attempting to steal all 10 paintings. What is the probability that she is successful?

$n = 10$ $k = 10$ $p = 0.05$ $1 - p = 0.95$		$n = 10$ $k = 0$ $p = 0.95$ $1 - p = 0.05$
$({}_{10}C_{10})(.05)^{10}(.95)^0$ <div style="border: 1px solid purple; border-radius: 50%; padding: 5px; display: inline-block;"> <math>9.77 \times 10^{-14}</math> </div>		$({}_{10}C_0)(.95)^0(.05)^{10}$ <div style="border: 1px solid purple; border-radius: 50%; padding: 5px; display: inline-block;"> <math>9.77 \times 10^{-14}</math> </div>

2) A drug company claims their new flu vaccine cures 92% of patients. What is the probability that in a family of five people who take the vaccine:

a) none get the flu?

$$n = 5$$

$$k = 0$$

$$p = .08 \quad 1 - p = .92$$

$$({}_5C_0)(.08)^0(.92)^5 = \text{0.66}$$

b) they all get the flu?

$$n = 5$$

$$k = 5$$

$$p = .08 \quad 1 - p = .92$$

$$({}_5C_5)(.08)^5(.92)^0 = \text{3.28} \times 10^{-6}$$

$$\text{.00000328}$$

c) at most 1 person gets the flu?

$$n = 5$$

$$k = 0 \text{ or } 1$$

$$p = .08$$

$$1 - p = .92$$

$$({}_5C_0)(.08)^0(.92)^5 + ({}_5C_1)(.08)^1(.92)^4$$

$$\text{0.95}$$

3) A manufacturer makes 10 000 ball point pens per day and estimates that 400 will be defective. What is the probability that in a random sample of 10 pens, 3 are defective?

$$n = 10$$

$$k = 3$$

$$p = \frac{400}{10000} = .04$$

$$1 - p = .96$$

$$({}_{10}C_3)(.04)^3(.96)^7$$

$$\text{.006}$$

4) About 35% of all U.S. households have a cellular phone. If you called 7 households, what's the probability that

a) every household had a cellular phone?

$$n=7$$
$$k=7$$

$$({}_7C_7)(.35)^7(.65)^0 = 6.43 \times 10^{-4}$$
$$.000643$$

$$p=.35 \quad 1-p=.65$$

b) exactly 4 households had a cellular phone?

$$n=7$$

$$k=4$$

$$p=.35 \quad 1-p=.65$$

$$({}_7C_4)(.35)^4(.65)^3 = 0.14$$

c) fewer than 2 households had a cellular phone?

$$n=7$$

$$k=0 \text{ or } 1$$

$$p=.35$$

$$1-p=.65$$

$$({}_7C_0)(.35)^0(.65)^7 + ({}_7C_1)(.35)^1(.65)^6$$

$$0.23$$

d) at least 6 households had a cellular phone?

$$n=7$$

$$k=6 \text{ or } 7$$

$$p=.35$$

$$1-p=.65$$

$$({}_7C_6)(.35)^6(.65)^1 + ({}_7C_7)(.35)^7(.65)^0$$

$$0.013$$

5) About 75% of all marketing personnel are extroverts. What's the probability that in a sample of 5, exactly 4 are extroverts?

$$n=5$$

$$k=4$$

$$p=.75$$

$$1-p=.25$$

$$({}_5C_4)(.75)^4(.25)^1 = 0.40$$

6) About 48% of all students get detention. What's the probability that in a sample of 6, only 2 have had detention?

$$n=6$$

$$k=2$$

$$p=0.48 \quad 1-p=.52$$

$$({}_6C_2)(.48)^2(.52)^4 = 0.25$$

7) A manufacturer of halogen bulbs knows that 3% of the production of their 100 W bulbs will be defective. What is the probability that exactly 5 bulbs in a carton of 144 bulbs will be defective?

$$n=144$$

$$k=5$$

$$p=.03$$

$$1-p=.97$$

$$({}_{144}C_5)(.03)^5(.97)^{139} = 0.17$$

8) A packet of carrot seeds has a germination rate of 92%. In other words, the probability of any seed sprouting is 0.92. How many seedlings would you expect in a row of 50 seeds?

$$(50)(0.92) = 46$$

9) A packet of vegetable seeds has a germination rate of 96%. What is the probability that exactly 10 of 12 seeds planted will sprout?

$$n=12 \quad (12C_{10})(.96)^{10}(.04)^2 = 0.07$$

$$k=10$$

$$p=.96$$

$$1-p=.04$$

10) The probability that an evening college student will graduate is 0.5. Determine the probability that out of 7 students, ...

a) none will graduate.

$$n=7$$

$$k=0$$

$$(7C_0)(0.5)^0(0.5)^7 = 0.008$$

$$p=0.5 \quad 1-p=0.5$$

b) all will graduate.

$$n=7$$

$$k=7$$

$$(7C_7)(0.5)^7(0.5)^0 = 0.008$$

$$p=0.5 \quad 1-p=0.5$$

c) one will graduate.

$$n=7$$

$$k=1$$

$$(7C_1)(0.5)^1(0.5)^6 = 0.05$$

$$p=0.5 \quad 1-p=0.5$$

d) at least 5 will graduate.

$$n=7$$

$$k=5, 6, \text{ or } 7$$

$$p=0.5$$

$$1-p=0.5$$

$$(7C_5)(0.5)^5(0.5)^2 + (7C_6)(0.5)^6(0.5)^1 + (7C_7)(0.5)^7(0.5)^0 = 0.23$$

11) A basketball player has a 60% chance of making each free throw. What is the probability that the player makes exactly three out of six free throws?

$$n=6$$

$$k=3$$

$$p=.6 \quad 1-p=.4$$

$$(6C_3)(.6)^3(.4)^3 = 0.28$$

12) A class has five students. What is the probability that exactly two of the students were born on a weekend?

$$n=5$$

$$k=2$$

$$p = \frac{2}{7} = .29 \quad 1-p = \frac{5}{7} = .71$$

$$(5C_2)(.29)^2(.71)^3$$

$$= 0.30$$