Name:________________________

Percentiles and Z-scores Worksheet
Please show all work.

1. **Shoes.** How many pairs of shoes do students have? Do girls have more shoes than boys? Here are data from a random sample of 20 female and 20 male students at a large high school:

Female	50	26	26	31	57	19	24	22	23	38
	13	50	13	34	23	30	49	13	15	51

Male	14	7	6	5	12	38	8	7	10	10
	10	11	4	5	22	7	5	10	35	7

a) Find the percentile in the female distribution for the girl with 22 pairs of shoes.

$\frac{6}{20} = .3 \rightarrow 30\% \rightarrow 30^{th}$ percentile

b) Find the percentile in the male distribution for the boy with 22 pairs of shoes.

$\frac{18}{20} = .9 \rightarrow 90\% \rightarrow 90^{th}$ percentile

c) Who is more unusual: the girl with 22 pairs of shoes or the boy with 22 pairs of shoes? Explain.

The boy → he has a higher percentile; 90% of boys have 22 pairs of shoes or less and only 30% of girls have 22 pairs of shoes or less

3. **WPM.** The average reading speed of students completing a speed-reading course is 450 words per minute (wpm). If the standard deviation is 70 wpm, find the z-scores associated with each of the following reading speeds.

a) 320 wpm

$\frac{320-450}{70} = -1.86$

b) 475 wpm

$\frac{475-450}{70} = 0.36$

c) 610 wpm

$\frac{610-450}{70} = -5.57$

4. **SAT vs. ACT**. Eleanor scores 680 on the SAT mathematics test. The distribution of SAT scores is symmetric and single peaked, with a mean of 500 and standard deviation of 100. Gerald takes the American College Testing (ACT) Mathematics test and scores 27. ACT scores also follow a symmetric, single-peaked distribution- but with mean 18 and standard deviation 6. Find the standardized scores (z-scores) for both students. Assuming that both tests measure the same kind of ability, who has the higher score?

Eleanor

$\frac{680-500}{100} = 1.8$

Gerald

$\frac{27-18}{6} = 1.5$

Both students scored above average but since Eleanor has a higher z-score we know she performed better when compared to others than Gerald

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5. **Comparing batting averages.** Three landmarks of baseball achievement are Ty Cobb's batting average of 0.420 in 1911, Ted William's 0.406 in 1941 and George Brett's 0.390 in 1980. These batting averages cannot be compared directly because the distribution of major league batting averages has changed over the years. The distributions follow a Normal curve, except for outliers such as Cobb, Williams and Brett. While the men batting average has been held roughly constant by rule changes and the balance between hitting and pitching, the standard deviation has dropped over time. Here are the facts:

Decade	Mean	Standard Deviation
1910s	**0.266**	**0.0371**
1940s	**0.267**	**0.0326**
1970s	**0.261**	**0.0317**

Compute the standardized batting averages (z-scores) for Cobb, Williams, and Brett to compare how far each stood above his peers.

Cobb

$\frac{.420-.266}{.0371} = 4.15$

Williams

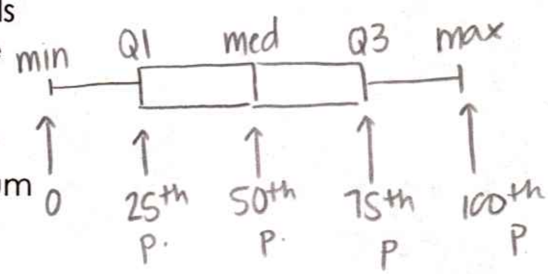
$\frac{.406-.267}{.0326} = 4.26$

Brett

$\frac{.390-.261}{.0317} = 4.07$

6. **Multiple Choice.** Jorge's score on Exam 1 in his statistics class was at the 64th percentile of the scores for all students. His score falls

a) between the minimum and the first quartile
b) between the first quartile and the median
c) between the median and the third quartile
d) between the third quartile and the maximum
e) at the mean score for all students



7. **Multiple Choice.** Scores on the ACT college entrance exam follow a Normal distribution with a mean 18 and a standard deviation 6. Wayne's standardized score (z-score) on the ACT was -0.7. What was Wayne's actual ACT score?

a) 4.2 b) -4.2 c) 13.8 d) 17.3 e) 22.2

$-0.7 = \frac{x-18}{6}$

$-4.2 = x - 18$

$13.8 = x$

9. **Midterm Exams.** Suppose that your chemistry professor returned your first midterm exam with only a z-score written on it. She told you that a histogram of the scores was closely described by a normal curve. How would you interpret each of the following z-scores?

a) 2.2 b) -0.4 c) -1.8 e) 0.0

a) well above average; really good

b) a little below average but not bad

c) below average; probably bad but not terrible

e) average

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10. A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

a) A score of 60, where the mean score of the sample data values is 40.

$$\frac{60-40}{10} = 2$$

b) A score that is 30 points below the mean 40.

$$\frac{10-40}{10} = -3$$

c) A score of 80, where the mean score of the sample data values is 30.

$$\frac{80-30}{10} = 5$$

d) A score of 20, where the mean score of the sample data values is 50.

$$\frac{20-50}{10} = -3$$

11. IQ scores have a mean of 100 and a standard deviation of 16. Albert Einstein reportedly had an IQ of 160.

a. What is the difference between Einstein's IQ and the mean?

$160 - 100 = 60$

b. Convert Einstein's IQ score to a z score.

$$\frac{160-100}{16} = 3.75$$

c. If we consider "usual IQ scores to be those that convert z scores between -2 and 2, is Einstein's IQ usual or unusual?

Albert Einstein's IQ would be considered unusually high

12. Three students take equivalent stress tests. Which is the highest relative score (meaning which has the largest z score value)?

a. A score of 144 on a test with a mean of 128 and a standard deviation of 34.

$$\frac{144-128}{34} = 0.47$$

b. A score of 90 on a test with a mean of 86 and a standard deviation of 18.

$$\frac{90-86}{18} = 0.22$$

c. A score of 18 on a test with a mean of 15 and a standard deviation of 5.

$$\frac{18-15}{5} = 0.6$$

$\boxed{\text{Student C}}$

Statistical Reasoning Name:
Chapter 3 Quiz Review

On a statistic test the class mean was 63 and the standard deviation was 7 and for the biology test the mean was 23 and has a standard deviation of 3.9.

1) Determine on which test the student had a better score. *(For each student you are determining whether they did better on the stats test or the biology test)*

i. A student received a 73 on the statistics test and a 26 on the biology test.

$\frac{73-63}{7} = 1.43$ $\frac{26-23}{3.9} = 0.77$

$\boxed{\text{Statistics}}$

ii. A student gets a 60 on the statistics tests and a 20 on the biology test.

$\frac{60-63}{7} = -0.43$ $\frac{20-23}{3.9} = -0.77$

$\boxed{\text{Statistics}}$

iii. A student gets a 78 on the statistics test and a 29 on the biology test.

$\frac{78-63}{7} = 2.14$ $\frac{29-23}{3.9} = 1.54$

$\boxed{\text{Statistics}}$

iv. A student gets a 63 on the statistics test and a 23 on the biology test.

$\frac{63-63}{7} = 0$ $\frac{23-23}{3.9} = 0$

$\boxed{\text{the same}}$

2) If a student had a z-score of 1.32 on the statistics test, what was their test score?

$1.32 = \frac{x-63}{7}$ $9.24 = x - 63$ $x = 72.24$

3) How can you tell from a z-score if a student scored above or below the class average?

If z-score is + : scored above average
If z-score is − : scored below average

The data set below represents weights of carry-on luggage (in pounds) for a random sample of passengers returning from a vacation to Hawaii. Use this data to answer the questions below.

3	12	12	14	17	18	18
18	19	19	21	21	21	22
26	26	26	27	27	28	29
29	30	31	32	32	32	33
35	36	36	38	38	40	41
42	45	47	47	48	50	54

1) What percentile is the person's whose luggage weighs 21 pounds in?

$\frac{13}{42} = .31 \rightarrow$ 31^{st} percentile

2) What percentile is the person's whose luggage weights 45 pounds in?

$\frac{37}{42} = .88 \rightarrow$ 88^{th} percentile

3) Which weight from the sample represents the 38th percentile?

$.38 = \frac{x}{42}$ $x = 15.96 \rightarrow$ Find 16^{th} # $\rightarrow$ 26 lbs

4) What additional information from the sample would we need to calculate z-scores for the individual luggage weights?

None → we can calculate the mean and standard deviation with what we are given

Give an example or when having a low z-score and low percentile would be best.

* golf
* running a race

Give an example of when having a high z-score and high percentile would be best.

* basketball
* test scores

The ages of 21 cars randomly selected in a student parking lot are shown below:

12 6 4 9 11 1 7 8 9 8

9 1 3 5 15 7 6 8 8 2 1 5

1) Find the mean age. Round to the nearest tenth if needed.

6.6

2) Find the median age. Round to the nearest tenth if needed.

7

3) Find the standard deviation of the ages. Round to the nearest hundredth if needed.

3.69

4) Find the z-score for a car that is 7 years old. Interpret what this value tells you.

$\frac{7-6.6}{3.69} = \boxed{.11}$ The car is older than average but not by much.

5) Find the z-score for the car that is 3 years old. Interpret what this value tells you.

$\frac{3-6.6}{3.69} = \boxed{-0.98}$ The car is newer than average but not unusually new

6) What would be the age of the car that has a z-score of -2.5?

$-2.5 = \frac{x-6.6}{3.69}$ $-9.225 = x - 6.6$ $x = -2.625$

can't have - yrs old

↳ the car is brand new

7) For the car that is 4 years old...

a) Determine the z-score. Does this tell you that the car is new or old compared to the others? Explain.

$\frac{4-6.6}{3.69} = -0.70$ New since z-score is negative the age is less than average age

b) Determine the percentile. Does this tell you that the car is new or old compared to the others? Explain.

$\frac{6}{22} = 0.27$ ↓ 27th percentile

The car is new; only 27% of cars are this age or newer