Statistical Reasoning Name:
5.2 – Margin of Error and All That

The **margin of error** that sample surveys announce translates sampling variability into a statement of how much confidence we can have in the results of a survey.

Often in the news, you hear statements such as "margin of error plus or minus 3 percentage points". This means that if we were to take many samples using the same method, 95% of the samples would give a result within plus or minus 3 percentage points of the truth about the population.

The margin of error tells us how close our estimate comes to the truth, but we can't be *certain* that the truth differs from the estimate by no more than the margin of error. 95% of all samples come close to the truth, but 5% miss by more than the margin of error. We don't know the truth about our population – but we know that 95% of our samples hit and 5% our samples miss. We say that we are **95% confident** that the truth lies within the margin of error.

A quick method for margin of error:

If you use the sample proportion $\hat{p}$ from a simple random sample of size n to estimate an unknown proportion p, then the margin of error for 95% confidence is roughly equal to $\frac{1}{\sqrt{n}}$.

sample size

A **confidence statement** has two parts: a **margin of error** and a **level of confidence**. The margin of error says how close the sample statistic lies to the population parameter. The level of confidence says what percent of all possible samples satisfy the margin of error. Here are some hints for interpreting confidence statements:

- The conclusion of a confidence statement always applies to the population, not the sample
- Our conclusion about the population is never completely certain
- A sample survey can choose to use a confidence level other than 95%
- It is usual to report the margin of error for 95% confidence
- A larger sample results in a smaller margin of error

Gallup began its poll on gambling with this question: "First, generally speaking, do you approve or disapprove of legal gambling or betting?" In addition to the 1523 adults (aged 18 or older) who lottery habits we have explored, Gallup took a random sample of 501 teenagers (aged 13 to 17). The sample results were:

Adults: 959 out of 1523 approve

Teens: 261 out of 501 approve

Find the 95% confidence interval for the true proportion of teenagers who approve of legal gambling or betting.

① sample proportion $\hat{p} = \frac{261}{501} = 0.521$

② m.o.e. $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{501}} = 0.045$

③ confidence interval = $\hat{p} \pm$ m.o.e.
$.521 \pm .045 \rightarrow (.476, .566)$

④ Interpret: We are 95% confident that the true proportion of all teens who approve of legal gambling is between .476 and .566.

The paralyzed Veterans of America is a philanthropic organization that relies on contributions. They send free mailing labels and greeting cards to potential donors on their list and ask for voluntary contribution. To test a new campaign they recently sent letters to a random sample of 100,000 potential donors and received 4781 donations.

a) Give a 95% confidence interval for the true proportion of those from their entire mailing list who may donate.

① $\hat{p} = \frac{4781}{100{,}000} = 0.048$

② $\text{moe} = \frac{1}{\sqrt{100{,}000}} = .003$

③ confidence interval

$.048 \pm .003 = (.045, .051)$

④ Interpret

We are 95% sure that the true proportion of potential donors who donate is between .045 and .051.

b) A staff member thinks that the true rate is 5%. Given the confidence interval you found, do you find that percentage plausible?

Yes, since 5% (.05) is in our confidence interval, it makes sense that it could be the true proportion/percent

Recently, ACT, Inc. reported that 71.9% (363) of 505 randomly selected college students in public colleges that were freshmen in 2010 returned to college in 2011 for their sophomore year.

Estimate the national freshman-to-sophomore retention rate in colleges with a public 95% confidence interval.

$\hat{p} = .719$

$\text{moe} = \frac{1}{\sqrt{505}} = .044$

$.719 \pm .044 = (.675, .763)$

We are 95% confident that the true proportion of college freshmen who return to college for their soph. year is between .675 and .763.

We are 95% confident that the true freshman-to-sophomore retention rate for public colleges is between .675

just need 1, not both

Statistical Reasoning Name: answer key
95% Confidence Intervals Practice

1) A random sample of 200 computer chips is obtained from one factory and 4% are $\hat{p}$ found to be defective. Construct and interpret a 95% confidence interval for the proportion of all computer chips from that factory that are defective. Round all decimals to the nearest thousandth.

$\hat{p} = .04$

$moe = \frac{1}{\sqrt{200}} = .071$

confidence interval = $.04 \pm .071$

$(-.031, .111)$

$(0, .111)$

We are 95% confident that the true proportion of defective computer chips is between 0 and 0.111

↑ confidence statement (interpretation)

2) Out of 54 randomly selected patients of a local hospital who were surveyed, 49 reported that they were satisfied with the care they received. Construct and interpret a 95% confidence interval for the percentage of all patients satisfied with their care at the hospital. Round all decimals to the nearest thousandth.

$\hat{p} = \frac{49}{54} = .907$

$moe = \frac{1}{\sqrt{54}} = .136$

confidence interval: $(.771, 1.043)$

$(.771, 1)$

confidence statement: We are 95% confident that the true proportion of patients who are satisfied with their care is between .771 and 1.

3) Wildlife biologists inspect 153 deer taken by hunters and find 32 of them carrying Lyme disease ticks.

a) Calculate a 95% confidence interval for the proportion of deer that carry Lyme disease ticks. Round all decimals to the nearest thousandth.

$\hat{p} = \frac{32}{153} = .209$

$moe = \frac{1}{\sqrt{153}} = .081$

confidence interval: (.128, .29)

confidence statement: We are 95% confident that the true proportion of deer carrying Lyme disease is between .128 and .29.

b) If the scientists want to cut the margin of error in half, how many deer must they inspect?

$\frac{.081}{2} = .0405$

$moe = \frac{1}{\sqrt{n}}$

$\sqrt{n}(.0405) = \left(\frac{1}{\sqrt{n}}\right)\sqrt{n}$

$\frac{.0405(\sqrt{n})}{.0405} = \frac{1}{.0405}$

$(\sqrt{n})^2 = (24.69)^2$

$n = 609.5961$

610 deers

4) EMC research conducted a poll between Jan. 14 and Jan 22 2014. They asked a SRS of 805 voting Seattlites if they supported the minimum raise hike to $15/hr. They found a sample proportion of 68%. What is the margin of error?

$\hat{p} = .68$

$moe = \frac{1}{\sqrt{805}} = .035$

confidence interval: (.645, .715)

confidence statement: We are 95% confident that the true proportion of Seattlites who favor raising minimum wage is between .645 and .715