Statistical Reasoning   Name:

Chapter 8 – Week 1 – Quiz Review

Part One: Expected Value

Find the expected value for the probability distributions shown in the tables below.

1)

Outcome, x	1000	500	-100
Probability, p	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{3}{8}$

$(1000)\left(\frac{1}{8}\right)+(500)\left(\frac{4}{8}\right)+(-100)\left(\frac{3}{8}\right)$

$=337.5$

2)

Outcome, x	0	10	15	20
Probability, p	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$(0)\left(\frac{2}{5}\right)+(10)\left(\frac{1}{5}\right)+(15)\left(\frac{1}{5}\right)+(20)\left(\frac{1}{5}\right)$

$=9$

3)

Outcome, x	-2	10	100
Probability, p	0.4	0.4	0.2

$(-2)(0.4)+(10)(0.4)+(100)(0.2)$

$=23.2$

example: Matt puts 3 nickels, 4 dimes, and 2 quarters into a bag. He then picks one coin at random. What is the expected value, in cents, of each pick in this experiment?

When Matt picks out one coin, it will either be a nickel, a dime or a quarter. Our outcomes must be numerical – so we can't list nickel, dime, quarter as the three outcomes. Instead, we can list the value in cents of each coins as the outcomes. Picking a nickel = picking 5¢. Picking a dime = picking 10¢. Picking a quarter = picking 25¢.

Outcome, x	5¢	10¢	25¢
Probability, p	$\frac{3}{9}=0.33$	$\frac{4}{9}=0.44$	$\frac{2}{9}=0.22$

Before you keep going, check and make sure the following are true:

All outcomes are numerical - ✔

All probabilities are between 0 and 1 - ✔

All probabilities add to 1 – if you add the decimals, you get 0.99 since we rounded – you can confirm it adds to 1 by adding the fractions. $\frac{3}{9}+\frac{4}{9}+\frac{2}{9}=1$ ✔

Expected value = $(5)\left(\frac{3}{9}\right)+(10)\left(\frac{4}{9}\right)+(25)\left(\frac{2}{9}\right)=\frac{35}{3}=11.67$¢ *or* 12¢

OR Expected value = $(5)(0.33)+(10)(0.44)+(25)(0.22)=11.55$¢ *or* 12¢

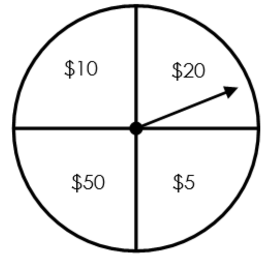
I am fine with you using the decimals or the fractions when finding expected value but please show your work!

4) Frank puts 10 pennies, 4 nickels, 3 dimes, and 3 quarters into a bag. He then picks one coin at random. What is the expected value, in cents, of each pick in this experiment?

Outcome, x	1¢	5¢	10¢	25¢
Probability, p	$\frac{10}{40}=0.25$	$\frac{4}{20}=0.2$	$\frac{3}{20}=0.15$	$\frac{3}{20}=0.15$

$ev=(1)(0.25)+(5)(0.2)+(10)(0.15)+(25)(0.15)=6.5$¢ *or* 7¢

5) At the county fair, you can play the spinner on the right. Every spin wins a prize. The section on which you land shows the value of the prize you win. What is the expected value of the amount of money you win each time you spin the spinner?

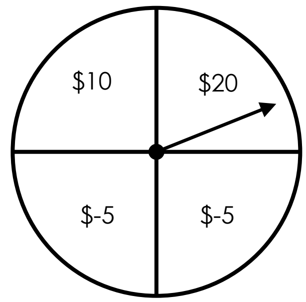


I like to order my outcomes from least to greatest – this is not mandatory

Outcome, x	$5	$10	$20	$50
Probability, p	$\frac{1}{4} = 0.25$	$\frac{1}{4} = 0.25$	$\frac{1}{4} = 0.25$	$\frac{1}{4} = 0.25$

$ev = (5)(0.25) + (10)(0.25) + (20)(0.25) + (50)(0.25) = 21.25$

6) At the county fair, you can play the spinner on the right. The section on which you land shows the amount of money you win or lose. Notice that if you play a game with this spinner you can lose $5. What is the expected value of the amount of money you win each time you spin the spinner? Would you pay $10 to spin this spinner?



Outcome, x	$ - 5	$ 10	$ 20
Probability, p	$\frac{2}{4} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{4} = 0.25$

$ev = (-5)(0.5) + (10)(0.25) + (20)(0.25) = \$5$

Whether or not you would pay $10 to spin the spinner is up to you – just make sure you provide a reason for your answer.

7) What is the expected value of rolling a die that has 6 sides? *(Hint: make a table to help you)*

x	1	2	3	4	5	6
P(x)	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$	$\frac{1}{6} = 0.17$

$ev = (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{6}\right) = 3.5 = \$3.50$

8) Jennifer is playing a game at an amusement park. There is a 0.1 probability that she will score 10 points, a 0.2 probability that she will score 20 points, and a 0.7 probability that she will score 30 points. How many points can Jennifer expect to receive by playing the game?

x	10	20	30
P(x)	0.1	0.2	0.7

$ev = (10)(0.1) + (20)(0.2) + (30)(0.7) = 26$ = 26 points

9) Mara is playing a game. There are two marbles in a bag. If she chooses the purple marble, she will win $10. If she chooses the orange marble, she will win $200. What is the expected value of Mara's winnings from the game?

x	$10	$200
P(x)	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$

$ev = (10)(0.5) + (200)(0.5) = 105$

Part Two: Probability Distributions

Consider each distribution. Determine if it is a valid probability distribution or not, and explain your answer. *(Hint: think about the checklist from the example worked out of the first page.)*

10)

x	0	1	2
P(x)	0.25	0.6	0.15

Yes – all outcomes are numerical ✓
each probability is between 0 and 1 ✓
sum of all probabilities is 1 ✓

11)

x	0	1	2
P(x)	0.25	0.60	0.2

No – all outcomes are numerical ✓
each probability is between 0 and 1 ✓
sum of all probabilities is NOT 1 X

USA Today reported that approximately 25% of all state prison inmates released on parole become repeat offenders while on parole. Suppose the parole board is examining five prisoners up for parole. Let x = number of prisoners out of five parole who become repeat offenders, and their corresponding probabilities.

x	0	1	2	3	4	5
P(x)	0.237	0.369	0.264	0.114	0.015	0.001

12) What is the probability that one or more of the five parolees will be repeat offenders?

$$0.369 + 0.264 + 0.114 + 0.015 + 0.001 = 0.736$$

13) Find the probability that two or more of the five parolees will be repeat offenders.

$$0.264 + 0.114 + 0.015 + 0.001 = 0.394$$

14) Find the probability that two or less of the five parolees will be repeat offenders.

$$0.237 + 0.369 + 0.264 = 0.87$$

15) Compute the mean number of repeat offenders out of five. *(mean = expected value in probability distributions)*

$$\bar{x} = ev = (0)(0.237) + (1)(0.369) + (2)(0.264) + (3)(0.114) + (4)(0.015) + (5)(0.001) = 1.304 \text{ or } 1 \text{ offender}$$

The random variable x is the number of houses sold by a realtor in a single month at the Sendsom's Real Estate office. Its probability distribution is as follows.

x	**P(x)**
0	0.24
1	0.01
2	0.12
3	0.16
4	0.01
5	0.14
6	0.11
7	0.21

16) What is the probability that a realtor sold less than 3 houses?

$$0.24 + 0.01 + 0.12 = 0.37$$

17) What is the probability that a realtor sold between 2 and 5 houses?

$$0.12 + 0.16 + 0.01 + 0.14 = 0.43$$

18) What is the probability that a realtor sold more than 6 houses?

$$0.21$$

19) What is the average number of houses sold?

$$\bar{x} = ev = (0)(0.24) + (1)(0.01) + (2)(0.12) + (3)(0.16) + (4)(0.01) + (5)(0.14) + (6)(0.11) + (7)(0.21) = 3.6 \text{ or } 4 \text{ houses}$$

Part Three: Counting Principles

Determine whether the following are examples of combinations or permutations. Then solve the problem.

20) A book club offers a choice of 8 books from a list of 40. In how many ways can a member make a selection?

combination 40 C 8 = 76,904,685 selections

21) How many different four-letter passwords can be created from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?

permutation 7 P 4 = 840 passwords

22) The school newspaper has an editor in chief and an assistant editor, The staff of the newspaper has 12 students. How many ways can two students be elected to the two positions?

permutation 23 P 2 = 132 ways

23) In a race, the top five finishers are awarded points for their team. There are 14 people running in a particular race. In how many ways can the runners finish in the top five?

combinations 14 C 5 = 2,002 ways

24) In a race, the top five finishers are awarded points for their team. There are 14 people running in a particular race. In how many ways can the runners finish 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, and 5^{th}?

permutation 14 P 5 = 240,240 ways

25) In order to conduct an experiment, four subjects are chosen randomly from a group of twenty subjects. How many different groups of four subjects are possible?

combination 20 C 4 = 4,845 groups

26) In how many ways can 8 people be seated in a row of 5 chairs?

permutation 8 P 5 = 6,720 ways

27) Five representatives from a class of 32 students are being chosen to be part of a committee. How many different ways are there for those five students to be chosen?

combination 32 C 5 = 201,376 ways

28) The Spanish Club is electing a president, vice president, and secretary from the 8 eligible members. How many different ways can these three positions be selected?

permutation 8 P 3 = 336 ways