Statistical Reasoning
Unit 3 Review: Modeling Distributions of Data

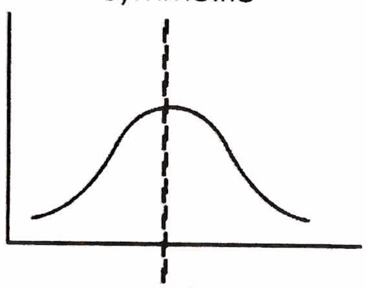
Name ____________________
Date ____________________

Describe and Interpret Data – shape, center, spread

- With skewed data, the ___mean___ is always pulled toward the skew.
- The ___median___ always divided the curve in half.
- The area under a density curve is always ___1___.

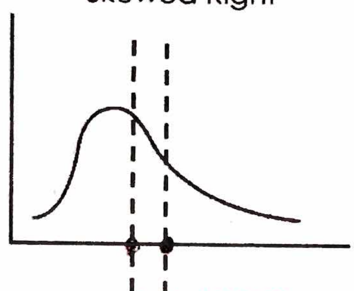
1. Identify which line represents the mean and which line represents the median for each distribution below. Then state the relationship between the median and mean for each distribution.

Symmetric



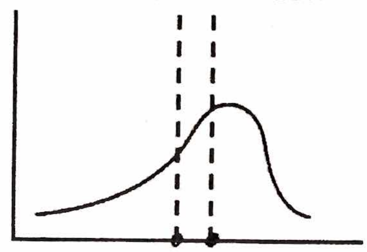
mean
median
mean ≈ median

Skewed Right



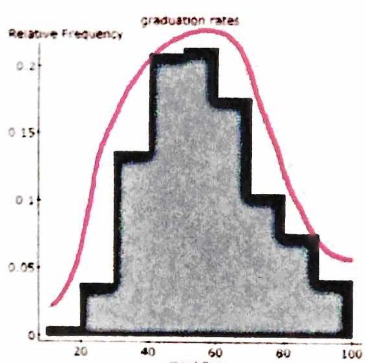
med. ↳ mean
mean > median

Skewed Left

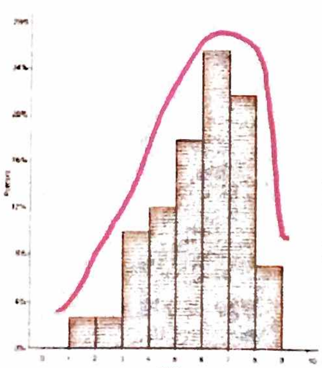


mean ↳ med.
mean < median

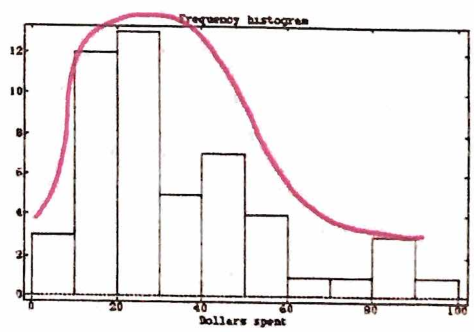
2. Sketch the density curve that best represents the histograms below. Describe each shape.



approximately symmetric



skewed left



skewed right

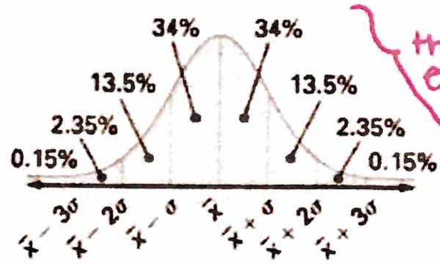
Statistical Reasoning
Unit 3 Review: Modeling Distributions of Data

Name ____________________
Date ____________________

Describe and interpret the patterns in variability for the distribution.

Key Facts:

- Not all data is normally distributed.
- Normal Distribution/Normal curve

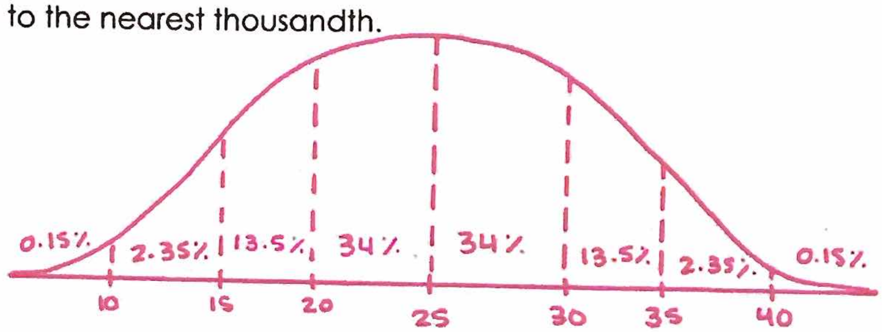


the empirical rule

- The standard Normal curve has z-scores on the x-axis with mean 0 and standard deviation of 1.
- Z-scores tell you how many standard deviations above / below the mean
    - Positive z-scores – above or below the mean?
    - Negative z-scores – above or below the mean?

standard normal { 0 → mean, 1 → standard deviation

1. A Normal distribution has a mean of 25 and a standard deviation of 5. Find the area under the curve for the following intervals. Record your answer as a decimal rounded to the nearest thousandth.



a. Between 20 and 30

68% → $\boxed{0.68}$

b. At least 20

↳ 20 and more

34 + 34 + 13.5 + 2.35 + .15 = 84

$\boxed{0.84}$

c. At most 30

↳ under 30

34 + 34 + 13.5 + 2.35 + .15 = 84

$\boxed{0.84}$

d. Less than 15 or more than 35

2.5 + 2.5

5% = $\boxed{0.05}$

.0018 = 0.18%

Statistical Reasoning
Unit 3 Review: Modeling Distributions of Data

Name ____________________
Date ____________________

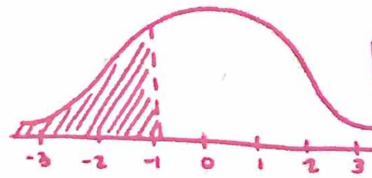
Use z-scores to calculate percentiles and areas under a Normal distribution

$$z = \frac{x - \overline{X}}{\sigma}$$

1. A Normal distribution has a mean of 112.8 and a standard deviation of 9.3. Use the **STANDARD** normal table to find the area covered by the following intervals:

   a. x ≤ 104.3

   $z = \frac{104.3 - 112.8}{9.3} = -0.91$

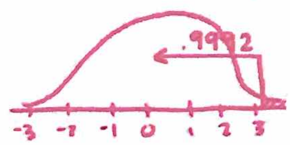
   

   $\boxed{0.1814}$

   b. x ≥ ~~[illegible]~~ 142

   $z = -3.57$ $\boxed{0}$ $\boxed{1}$

   $z = \frac{142 - 112.8}{9.3} = 3.14$

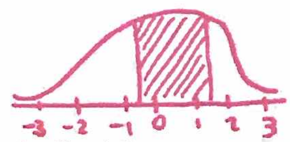
   

   $1 - .9992 = \boxed{.0008}$

   c. x is between 106.2 and 127.1

   $z = \frac{106.2 - 112.8}{9.3} = -0.71$

   $z = \frac{127.1 - 112.8}{9.3} = 1.54$

   $.9382 - .2389 = \boxed{.6993}$

2. Use the table given in class and the value of z to find the percentile.

   a. z = -1.78 → 0.0375 = 3.75% = $\boxed{\text{4th percentile}}$

   b. z = 2.62 → .9956 = 99.56% = $\boxed{\text{100th percentile}}$

   c. z = 0.34 → .6331 = 63.31% = $\boxed{\text{63rd percentile}}$

3. Scores on the SAT follow a normal distribution with mean 452 and standard deviation 18. Michael's z-score is 1.34; what is his actual SAT score?

   $1.34 = \frac{x - 452}{18}$ x = 476.12 or 476

4. What is the meaning of a z-score of -1?

   The data point is 1 standard deviation below the mean.

5. A z-score of 2.5 is equivalent to what percentile?

   .9938 = 99th percentile

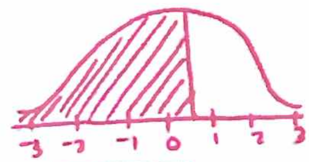
Statistical Reasoning Name ________________________

Unit 3 Review: Modeling Distributions of Data Date ________________________

extra information

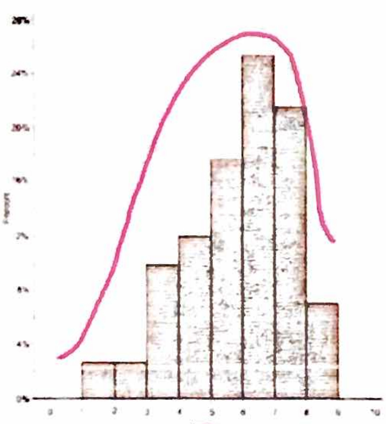
6. **Airport Temperatures.** The temperature is recorded at ~~60~~ airports in a region. The average temperature is 65 degrees Fahrenheit with standard deviation of 5 degrees. What is the probability that the temperature at a randomly selected airport is no more than 68 degrees?

$z = \frac{68-65}{5} = 0.6$ → less than or = to 68



$\boxed{.7257}$

A county is having a bottle recycling competition between homerooms in high schools around the county. The histogram below shows the percentage of homerooms that collected the amount of bottles collected in hundreds.



7. Draw a density curve for summarizing the histogram and describe the shape.

skewed left

8.Compare the mean and the median of the distribution.

mean < median

Statistical Reasoning
Unit 3 Review: Modeling Distributions of Data

Name ____________________
Date ____________________

1. A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

$z = \frac{x - \bar{x}}{s} = \frac{20}{10} = 2$

   a) A score that is 20 points above the mean. $z = 2$

   b) A score that is 10 points below the mean. $z = -1$

   c) A score that is 15 points above the mean $z = 1.5$

   d) A score that is 30 points below the mean. $z = -3$

2. The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have a mean of 35 and a standard deviation of 6. Assuming these raw scores form a normal distribution:

   a) What score represents the 65th percentile (what number separates the lower 65% of the distribution)?

   $.65 \rightarrow .6517 \rightarrow z = 0.39$ $\quad 0.39 = \frac{x - 35}{6}$ $\quad \boxed{37.34}$

   b) What score represents the 90th percentile?

   $.90 \rightarrow .9015 \rightarrow z = 1.29$ $\quad 1.29 = \frac{x - 35}{6}$ $\quad \boxed{42.74}$

   c) What percent of students get a raw score between $\boxed{28}$ and $\boxed{38}$?

   $z = \frac{28 - 35}{6} = -1.17 \rightarrow .1210$

   $z = \frac{38 - 35}{6} = 0.5 \rightarrow .6915$

   $.6915 - .1210$

   $\boxed{.5705}$ $\boxed{57.05\%}$

   d) What is the probability of getting a raw score between 41 and 44?

   $z = \frac{41 - 35}{6} = 1 \rightarrow .8413$

   $z = \frac{44 - 35}{6} = 1.5 \rightarrow .9332$

   $.9332 - .8413 = \boxed{.0919}$

Statistical Reasoning
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Name ______________________
Date ______________________

3. Scores on the SAT form a normal distribution with $\mu = 500$ and $\sigma = 100$.
   a) What is the minimum score necessary to be in the top 15% of the SAT distribution?

   85% → .85 → .8508 → 1.04

   $1.04 = \frac{x - 500}{100}$ $x = \boxed{604}$

   ~~b)~~ Find the range of values that defines the middle 80% of the distribution of SAT scores (372 and 628).

4. For a normal distribution, find the z-score that separates the distribution as follows:
   a) Separate the highest 30% from the rest of the distribution.

   .70 → .7019 → $\boxed{z = 0.53}$

   b) Separate the lowest 40% from the rest of the distribution.

   .40 → .4013 → $\boxed{z = -0.25}$

   c) Separate the highest 75% from the rest of the distribution.

   .25 → .2514 → $\boxed{z = -0.67}$

5. For the numbers below, find the area between the mean and the z-score:
   (remember the mean has a z-score of 0)

   → z = 0

   a) z = 1.17 → .8790   .8790 − .5000 = $\boxed{.379}$

   b) z = -1.37 → .0853   .5000 − .0853 = $\boxed{.4147}$

6. For the z-scores below, find the percentile rank (percent of individuals scoring below):
   a) -0.47 → .3192 → 31.92% → 32nd percentile

   b) 2.24 → .9875 → 98.75% → 99th percentile

Statistical Reasoning

Unit 3 Review: Modeling Distributions of Data

Name ______________________

Date ______________________

7. For the z-scores below, find the percent of cases falling above the z-score:

   a) 0.24 → .5948 $1-.5948 = .4052 = \boxed{40.52\%}$

   b) -2.07 → .0192 $1-.0192 = .9808 = \boxed{98.08\%}$

8. A patient recently diagnosed with Alzheimer's disease takes a cognitive abilities test and scores a 45. The mean on this test is 52 and the standard deviation is 5. What is the patient's percentile rank?

$\frac{45-52}{5} = -1.4 \rightarrow .0808 = 8.08\% = 8^{th}$ percentile

9. A fifth grader takes a standardized achievement test (mean = 125, standard deviation = 15) and scores a 148. What is the child's percentile rank?

$\frac{148-125}{15} = 1.53 \rightarrow .9370 = 93.7\% = 94^{th}$ percentile

10. Pat and Chris both took a spatial abilities test (mean = 80, std. dev. = 8). Pat scores a 76 and Chris scored a 94. What percent of individuals would score between Pat and Chris?

$\frac{76-80}{8} = -0.5 \rightarrow .3085$

$\frac{94-80}{8} = 1.75 \rightarrow .9599$

$.9599 - .3085 = .6514$

$\boxed{65.14\%}$

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Unit 3 Review: Modeling Distributions of Data

Name ______________________
Date ______________________

11. A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

a) A score of 60, where the mean score of the sample data values is 40.

$$\frac{60-40}{10} = 2$$

b) A score that is 30 points below the mean.

$$\frac{-30}{10} = -3$$

c) A score of 80, where the mean score of the sample data values is 30.

$$\frac{80-30}{10} = 5$$

d) A score of 20, where the mean score of the sample data values is 50.

$$\frac{20-50}{10} = -3$$

12. IQ scores have a mean of 100 and a standard deviation of 16. Albert Einstein reportedly had an IQ of 160.

a. What is the difference between Einsteins IQ and the mean?

60 points

b. How many standard deviations is that?

$$\frac{60}{16} = 3.75$$

c. Convert Einstein's IQ score to a z score.

$$\frac{160-100}{16} = 3.75$$

d. If we consider "usual IQ scores to be those that convert z scores between -2 and 2, is Einstein's IQ usual or unusual?

unusually high

Statistical Reasoning
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Name ______________________

Date ______________________

13. Women's heights have a mean of 63.6 in. and a standard deviation of 2.5 inches. Find the z score corresponding to a woman with a height of 70 inches and determine whether the height is unusual.

$z = \frac{70 - 63.6}{2.5} = 2.56$

Her height is unusually tall.

14. Three students take equivalent stress tests. Which is the highest relative score (meaning which has the largest z score value)?

a. A score of 144 on a test with a mean of 128 and a standard deviation of 34.

$\frac{144 - 128}{34} = 0.47$

b. A score of 90 on a test with a mean of 86 and a standard deviation of 18.

$\frac{90 - 86}{18} = 0.22$

c. A score of 18 on a test with a mean of 15 and a standard deviation of 5.

$\frac{18 - 15}{5} = 0.6$

__C__ has the highest score.