Statistical Reasoning Name:
5.2 – Margin of Error and All That

The **margin of error** that sample surveys announce translates sampling variability into a statement of how much confidence we can have in the results of a survey.

Often in the news, you hear statements such as "margin of error plus or minus 3 percentage points". This means that if we were to take many samples using the same method, 95% of the samples would give a result within plus or minus 3 percentage points of the truth about the population.

The margin of error tells us how close our estimate comes to the truth, but we can't be *certain* that the truth differs from the estimate by no more than the margin of error. 95% of all samples come close to the truth, but 5% miss by more than the margin of error. We don't know the truth about our population – but we know that 95% of our samples hit and 5% our samples miss. We say that we are **95% confident** that the truth lies within the margin of error.

A quick method for margin of error:

If you use the sample proportion $\hat{p}$ from a simple random sample of size n to estimate an unknown proportion p, then the margin of error for 95% confidence is roughly equal to $\frac{1}{\sqrt{n}}$.

↖ sample size

A **confidence statement** has two parts: a **margin of error** and a **level of confidence**. The margin of error says how close the sample statistic lies to the population parameter. The level of confidence says what percent of all possible samples satisfy the margin of error. Here are some hints for interpreting confidence statements:

- The conclusion of a confidence statement always applies to the population, not the sample
- Our conclusion about the population is never completely certain
- A sample survey can choose to use a confidence level other than 95%
- It is usual to report the margin of error for 95% confidence
- A larger sample results in a smaller margin of error

Gallup began its poll on gambling with this question: "First, generally speaking, do you approve or disapprove of legal gambling or betting?" In addition to the 1523 adults (aged 18 or older) who lottery habits we have explored, Gallup took a random sample of 501 teenagers (aged 13 to 17). The sample results were:

Adults: 959 out of 1523 approve

Teens: 261 out of 501 approve

Adults

$\frac{959}{1523} = .63 = 63\%$ ← $\hat{p}$

$moe = \frac{1}{\sqrt{1523}} = .025 \rightarrow 3\%$ margin of error

① Find $\hat{p}$
② Find m.o.e.
③ Add/Sub. to get interval
④ Write statement

Teens

$\hat{p} = \frac{261}{501} = 52\%$

$moe = \frac{1}{\sqrt{501}} = 4\%$

interval: $52 - 4 = 48$
$52 + 4 = 56$

$63\% - 3\% = 60\%$
$63 + 3\% = 66\%$

We are 95% confident that between 60-66% of adults approve of legal gambling/betting.

Statement: We are 95% confident that between 48% and 56% of teens approve of legal gambling & betting

The paralyzed Veterans of America is a philanthropic organization that relies on contributions. They send free mailing labels and greeting cards to potential donors on their list and ask for voluntary contribution. To test a new campaign they recently sent letters to a random sample of 100,000 potential donors and received 4781 donations.

a) Give a 95% confidence interval for the true proportion of those from their entire mailing list who may donate.

$\hat{p} = \frac{4781}{100{,}000} = .05 = 5\%$

$moe = \frac{1}{\sqrt{100{,}000}} = .003 = 0.3\%$

Interval: $5 - .3 = 4.7$

$5 + .3 = 5.3$

$4.7\% - 5.3\%$

$(4.7\%, 5.3\%)$

$(.047, .053)$

b) A staff member thinks that the true rate is 5%. Given the confidence interval you found, do you find that percentage plausible?

Yes → 5% is in the interval (it is right in the middle)

Recently, ACT, Inc. reported that 71.9% (363) of 505 randomly selected college students in public colleges that were freshmen in 2010 returned to college in 2011 for their sophomore year.

a. Estimate the national freshman-to-sophomore retention rate in colleges with a public 95% confidence interval.

$\hat{p} = 71.9\%$

$moe = \frac{1}{\sqrt{505}} = 4\%$

Interval: $71.9 - 4 = 67.9\%$

$71.9 + 4 = 75.9\%$

OR $(67.9\%, 75.9\%)$

$(.679, .759)$

b. Choose the correct interpretation of the 95% confidence interval in part a:

i) In 95% of all random samples of public colleges, the freshman-to-sophomore retention rate will be 71.9%.

ii) In 95% of all random samples of public colleges, the freshman-to-sophomore retention rate is between 68.0% and 75.8%.

iii) There is a 95% chance that the interval (68.0%, 75.8%) contains the true freshmanto-sophomore retention rate for public colleges.

iv) There is a 95% chance that the true freshman-to-sophomore retention rate of public colleges is in the interval (68.0%, 75.8%)

v) If many random samples are selected, each sample with 505 public college freshmen, 95% of the sample freshman-to-sophomore retention rates will be in the interval: (.680, .758).